The Neumann problem for the fourth order improperly elliptic equations

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It will be investigated the following Neumann problem for the fourth order improperly elliptic equations in unit disk $K \subset R^2$:

$$L(\partial_x)u = a_0 \frac{\partial^4 u}{\partial x_1^4} + a_1 \frac{\partial^4 u}{\partial x_1^3 \partial x_2} + a_2 \frac{\partial^4 u}{\partial x_1 \partial x_2^3} + a_4 \frac{\partial^4 u}{\partial x_2^4} = 0, \tag{1}$$

$$u''_{\nu\nu} \mid_{\partial K} = 0, \; u'''_{\nu\nu\nu} \mid_{\partial K} = 0. \tag{2}$$

Here $\vec{\nu}-$ is the unit vector of outer normal, $\partial_x = (\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})$, $a_k \in \mathbb{C}$, $k = 0, 1, ..., 4$.

In the present report it will be obtained the conditions of nontriviality of the kernel and it will be shown (by investigating of these obtained conditions), that some classes of improperly elliptic equation have finite dimensional kernel (even trivial) that is these classes are analogue of properly elliptic equations, but some classes of improperly elliptic equations have infinitely dimensional kernel.

Theorem 1. *Let equation (1) be improperly elliptic under conditions*

$$\mathrm{Im}\lambda_1 > 0, \mathrm{Im}\lambda_2 > 0, \mathrm{Im}\lambda_3 > 0, \mathrm{Im}\lambda_4 < 0, \tag{3}$$

for roots of characteristics equation $L(1, \lambda) = 0$.

Then the Neumann problem (1), (2) has nontrivial kernel if and only if the following condition is true

$$\Delta_n = \det \begin{pmatrix} 1 & 1 & 1 & \mu_4^n \\ \mu_1 & \mu_2 & \mu_3 & \mu_4^{n-1} \\ \mu_1^{n-1} & \mu_2^{n-1} & \mu_3^{n-1} & \mu_4 \\ \mu_1^n & \mu_2^n & \mu_3^n & 1 \end{pmatrix} = 0, \tag{4}$$

for some $n > 3$. Here μ_j, $j = 1, ..., 4$, are determined by $\mu_1 = -\frac{\lambda_1 - i}{\lambda_1 + i}$, $\mu_2 = -\frac{\lambda_2 - i}{\lambda_2 + i}$, $\mu_3 = -\frac{\lambda_3 - i}{\lambda_3 + i}$, $\mu_4 = -\frac{\lambda_4 + i}{\lambda_4 - i}$.

Analogous result can be obtained for the second class of improperly elliptic fourth order equations:

$$\mathrm{Im}\lambda_1 > 0, \mathrm{Im}\lambda_2 > 0, \mathrm{Im}\lambda_3 > 0, \mathrm{Im}\lambda_4 > 0.$$