Approximation of boundary value problem solutions for integro-differential equations with delay

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We consider the following boundary value problem

$$y''(x) = f(x, [y(x)]) + \int_a^b g(x, t, [y(x)])dt, \ x \in [a; b] \tag{1}$$

$$y^{(i)}(x) = \varphi^{(i)}(x), \ i = 0, 1, \ x \in [a^*; a], \ y(b) = \gamma, \tag{2}$$

$$[y(x)] = (y(x), y(x - \tau(x)), y'(x), y'(x - \tau(x))), a^* = \min_{x \in [a;b]} (x - \tau(x)).$$

Existence and uniqueness of a boundary value problem with delay solution were studied in [1, 2]. Analytical solution of the problem (1)-(2) is possible only in the simplest cases. Spline collocation method usage for solving differential-difference equations was investigated in [3, 4]. In this paper an algorithm for finding the approximate solution of the boundary value problem (1)-(2) using cubic splines with defect two is suggested. Sufficient conditions for the iterative process convergence are obtained and numerical simulations to test cases are conducted.

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