Linear widths of the Nikol'skii-Besov classes

Nadya Derevyanko

Institute of Mathematics of NASU, Kyiv, Ukraine, nadyaderevyanko@gmail.com.

The talk concern the exact order estimates of linear widths of the Nikol'skii-Besov classes $B^r_{p,\theta}$ [1] of periodic functions of many variables in the space $L_q(\pi_d)$ for some relations between parameters p and q.

Let $F \subset L_q(\pi_d)$ is a certain class of function. Then linear widths of F in the space $L_q(\pi_d)$ is defined by [2]

$$\lambda_m(F, L_q) = \inf_{L^m \in \mathrm{Lin}_m(L_q)} \inf_{A \in \mathcal{L}(L_q, L^m)} \sup_{f \in F} \|f - Af\|_q,$$

where the inf is taken over all continuous linear operators A acting from L_q to L^m and over all such linear subspaces $L^m \subset L_q$ of dimension no larger than m.

Theorem 1. *Let $1 \le \theta \le \infty$. Then the following relation is true*

$$\lambda_m(B^r_{p,\theta}, L_q) \asymp \begin{cases} m^{-r/d+1/p-1/2}, & 1 \le p < 2 \le q < p'; r > d/p, \\ m^{-r/d+1/2-1/q}, & 1 < p \le 2,\ p' < q < \infty; r > d(1-1/q), \\ m^{-r/d+(1/p-1/q)_+}, & 2 \le q \le p \le \infty, 1 \le p < q \le 2, \\ & 2 \le p < q < \infty; r > d(1/p-1/q)_+, \end{cases}$$

where $a_+ = \max\{a, 0\}$, $\frac{1}{p} + \frac{1}{p'} = 1$.

[1] O. V. Besov, On some families of functional spaces. Imbedding and extension theorems. (Russian) *Dokl. Akad. Nauk SSSR*, **126** (1959), p. 1163–1165.

[2] V. M. Tikhomirov, Diameters of sets in functional spaces and the theory of best approximations. (Russian) *Uspehi Mat. Nauk*, **15** (1960), p. 81–120.