Approximation of the classes $S^r_{p,\theta}B(\mathbb{R}^d)$ by entire functions of a special form

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In the present talk, we discuss the problem of the approximation of the classes $S^{\boldsymbol{r}}_{p,\theta}B(\mathbb{R}^d)$ [1] of functions of many variables which are called Nikol'skii–Besov classes.

Consider a set $\mathfrak{M}$ formed by the vectors $\boldsymbol{s} = (s_1, \dots, s_d)$, $s_j \in \mathbb{Z}_+$, $j = \overline{1,d}$, and $Q_{2^{\boldsymbol{s}}} := \left\{\lambda \in \mathbb{R}^d : \ \eta(s_j)2^{s_j-1} \leqslant |\lambda_j| < 2^{s_j}, \ \ j = \overline{1,d}\right\}$, where $\eta(0) = 0$ and $\eta(t) = 1$, $t > 0$.

For $f \in L_q(\mathbb{R}^d)$, $1 < q < \infty$, we set $S^{\mathfrak{M}}(f, \boldsymbol{x}) = \sum_{\boldsymbol{s}\in\mathfrak{M}} \delta^*_{\boldsymbol{s}}(f, \boldsymbol{x})$, where $\delta^*_{\boldsymbol{s}}(f, \boldsymbol{x}) = \mathfrak{F}^{-1}(\chi_{Q_{2^{\boldsymbol{s}}}} \cdot \mathfrak{F}f)$, $\chi_{Q_{2^{\boldsymbol{s}}}}$ is the characteristic function of the set $Q_{2^{\boldsymbol{s}}}$, $\mathfrak{F}f$ and $\mathfrak{F}^{-1}f$ respectively direct and the inverse Fourier transform of the function f.

Further, for $f \in L_q(\mathbb{R}^d)$ we consider the approximative characteristic

$$e^{\mathfrak{F}}_M(f)_q = \inf_{\mathfrak{M}:\mathrm{mes}\bigcup_{\boldsymbol{s}\in\mathfrak{M}} Q_{2^{\boldsymbol{s}}} \leqslant M} \|f(\cdot) - S^{\mathfrak{M}}(f, \cdot)\|_q,$$

where mesA is the Lebesgue measure of the set A.

Theorem 1. *Let $1 < p < \infty$, $r_1 > \frac{1}{p}$. Then for $1 \leqslant \theta \leqslant \infty$ the order relation is true*

$$e^{\mathfrak{F}}_M(S^{\boldsymbol{r}}_{p,\theta}B)_\infty = \sup_{f\in S^{\boldsymbol{r}}_{p,\theta}B} e^{\mathfrak{F}}_M(f)_\infty \asymp \left(M^{-1}\log^{\nu-1} M\right)^{r_1-\frac{1}{p}} \left(\log^{\nu-1} M\right)^{1-\frac{1}{\theta}}.$$

[1] P. I. Lizorkin, S. M. Nikol'ski. Function spaces of mixed smoothness from the decomposition point of view, *Trudy Mat. Inst. Steklov*, **187** (1989), p. 143–161.