Approximation of periodic functions by analogue of Zigmund sums in Lebesgue spaces with variable exponents

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In this paper we consider the quantities of upper bounds

$$\mathcal{E}_n(L^{\psi}_{\beta,p(\cdot)})_{s(\cdot)} := \sup\Big\{\|f(\cdot) - \hat{Z}(f;\cdot)\|_{s(\cdot)} : \quad f \in L^{\psi}_{\beta,p(\cdot)}\Big\}$$

of deviations of analogue of Zigmund sums

$$\hat{Z}_n(f;x) := \frac{a_0(f)}{2} + \sum_{k=1}^{\infty}\Big(1 - \frac{\psi(n)}{\psi(k)}\Big)(a_k(f)\cos kx + b_k(f)\sin kx),$$

on the clases $L^{\psi}_{\beta,p(\cdot)}$ functions, whose $(\psi;\beta)$-derivatives belong to the unit ball of Lebesgue space $L^{p(\cdot)}$ with variable exponents. See [1, 2].

Theorem 1. *Let $p, s \in \mathcal{P}^{\gamma}$ and $s(x) \le p(x)$ on the period $x \in [0; 2\pi]$. If $\psi \in \Upsilon_n$, then for an arbitrary $n \in \mathbb{N}$ there exist positive constants $C_{p,s}$ and $K_{p,s}$, such that the following relation is true:*

$$C_{p,s}\psi(n) \le \mathcal{E}_n(L^{\psi}_{\beta,p(\cdot)})_{s(\cdot)} \le K_{p,s}\psi(n).$$

[1] A.I. Stepanets, *Methods of Approximation Theory.* VSP, Leiden–Boston, 2005.

[2] Diening L., Harjulehto P., Hästö P., Růžička M. *Lebesgue and Sobolev spaces with variable exponents*, SPIN, 2010.