Approximative properties of diagonal operators in Orlicz space

Andrii Shidlich[1], **Stanislav Chaichenko**[2]

[1]*Institute of Mathematics of NASU, Kyiv, Ukraine, andy709@list.ru;*
[2]*Donbas State Pedagogical University, Slavyansk, Ukraine, stolch@mail.ru*

Let $M(t)$, $t \geq 0$, be an Orlicz function, that is a non-decreasing convex down function such that $M(0) = 0$ and $M(t) \to \infty$ as $t \to \infty$. Orlicz sequence space l_M, defined by the function $M(t)$, is the linear space of all sequences $x = \{x_k\}_{k=1}^{\infty}$ of real numbers such that $\sum_k M(x_k) < \infty$. Equipped with the norm $||\mathbf{x}||_{l_M} := \inf\{\alpha > 0 : \sum_{k=1}^{\infty} M(|x_k|/\alpha) \leq 1\}$ it is a Banach space.

Let $\lambda = \{\lambda_k\}_{k=1}^{\infty}$ be an arbitrary bounded sequence of the positive numbers and let T: $x = \{x_k\}_{k=1}^{\infty} \to Tx = \{\lambda_k x_k\}_{k=1}^{\infty}$ be a diagonal operator defined on the space l_M.

Following S.B. Stechkin, for any sequence $x \in l_M$, consider the quantity $\sigma_n(x, l_M)$ of its best n-terms approximation, which is given by the relation

$$\sigma_n(x, l_M) := \inf_{\gamma_n} E_{\gamma_n}(x, l_M) = \inf_{a_i, \gamma_n} ||x - P_{\gamma_n}||_{l_M} = \inf_{a_i, \gamma_n} ||x - \textstyle\sum_{i \in \gamma_n} a_i e_i||_{l_M},$$

where γ_n are the arbitrary collections of n positive integers and $a_i \in \mathbb{R}$.

Theorem 1. *Assume that $0 < p < \infty$ is an arbitrary positive number and $M(t)$ is the Orlicz function such that $M(t^{1/p})$ is also the Orlicz function. Let also $\lambda = \{\lambda_k\}_{k=1}^{\infty}$ be an arbitrary non-increasing sequence of the positive numbers, satisfying condition $\lim_{k\to\infty} \lambda_k = 0$. Then for any $n \in \mathbb{N}$, the following equality is true:*

$$\sigma_n(T : l_p \to l_M) := \sup_{x \in Bl_p} \sigma_n(Tx, l_M) = \sup_{s > n} \left(\textstyle\sum_{k=1}^{s} \lambda_k^{-p}\right)^{-\frac{1}{p}} \left(M^{-1}(1/(s-n))\right)^{-1}, \quad (1)$$

where M^{-1} is the inverse function of M. The least upper bound on the right-hand side of (1) is attained at some finite value s^* of s.