On approximation of periodic functions in Hölder spaces

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Let $\mathcal{L}_n(f) = f * K_n$, where $f \in L_p = L_p(0, 2\pi)$, $1 \le p \le \infty$, and $K_n(x) = \sum_\nu a_{\nu,n} e^{\nu k x}$, where $a_{0,n} = 1$ and $a_{\nu,n} = 0$ for any $|\nu| > n$ and $n \in \mathbb{N}$.

In last years, there has been some interest in investigating of approximation of functions by such means in Hölder spaces $H_p^{r,\alpha}$. We will say that $f \in H_p^{r,\alpha}$, if $f \in L_p$ and

$$\|f\|_{H_p^{r,\alpha}} = \|f\|_p + \sup_{h>0} \frac{\|\Delta_h^r f\|_p}{h^\alpha} < \infty,$$

where $\Delta_\delta^r f(x) = \sum_{\nu=0}^r (-1)^\nu \binom{r}{\nu} f(x + \nu\delta)$.

One of our main results is as follows.

Theorem 1. *Let $1 \le p \le \infty$, $r \ge \alpha \ge 0$, and $r \in \mathbb{N}$. Let $\{\mathcal{L}_n\}$ be bounded in L_p and $w_\mathcal{L}$ is such that $w_\mathcal{L}(f, \delta)_p \le C\|f\|_p$ and $w_\mathcal{L}(f+g, \delta)_p \le C(\mathcal{L}, p)(w_\mathcal{L}(f, \delta)_p + w_\mathcal{L}(g, \delta)_p)$ for any $f, g \in L_p$ and any $\delta > 0$. Suppose that for any $f \in L_p$ and $n \in \mathbb{N}$ the following equivalence holds*

$$\|f - \mathcal{L}_n(f)\|_p \asymp w_\mathcal{L}(f, 1/n)_p.$$

Then for $f \in H_p^{r,\alpha}$ and $n \in \mathbb{N}$ we have

$$\|f - \mathcal{L}_n(f)\|_{H_p^{r,\alpha}} \asymp w_\mathcal{L}(f, 1/n)_p + \sup_{h>0} \frac{w_\mathcal{L}(\Delta_h^r f, 1/n)_p}{h^\alpha},$$

where $\asymp$ means a two-sided inequality with positive constants independent of f and n.

Similar results are proved also for approximation of functions in the spaces L_p, $0 < p < 1$.